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$|\Delta I| = 1/2$ Rule in the Standard Model

Mark A. Samuel[1]

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We consider the $K \to \pi\pi$ amplitudes in the Standard Model. We show that the infamous $|\Delta I| = 1/2$ rule can be explained by using Padé approximants to sum the diverging QCD perturbation series.

INTRODUCTION

It has been known for over 30 years that the $K \to \pi\pi$ decay amplitudes obey an approximate $|\Delta I| = 1/2$ selection rule. The $|\Delta I| = 1/2$ amplitude is much larger than the $|\Delta I| = 3/2$ amplitude. Although the $|\Delta I| = 3/2$ amplitude is explained in the Standard Model, theoretical calculations fail to reproduce the observed enhancement in $|\Delta I| = 1/2$ amplitudes of K decays by more than an order of magnitude.

In fact, Pich *et al.* (1986) conclude: "Our conclusion that $|\Delta I| = 1/2$ transitions in K-decays pose a serious problem to a 'natural' understanding within the framework of the Standard Model. We find a serious discrepancy which in order to be solved requires in our opinion a rather subtle mechanism in the strong interaction dynamics, or perhaps, new physics."

Although Stech (1991) has claimed to have resolved the problem, his calculation involves diquark–antidiquark operators in a phenomenological model and has not been accepted by the physics community (Jamir and Pich, 1994). Our calculations, however, are strictly within the Standard Model.

Quantitatively the situation is described in terms of the coupling constants as follows (Pich *et al.*, 1986):

$$[g^{(1/2)}]_{\text{exp}} = [g_8^{(1/2)} + g_{27}^{(1/2)}]_{\text{exp}} = 5.1 \tag{1}$$

$$[g_{27}^{(3/2)}]_{\text{exp}} = 0.16 \tag{2}$$

[1] Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078, and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309; e-mail: physmas@mvs.ucc.okstate.edu.

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The theoretical estimates obtained in Pich *et al.* (1986) are

$$[g_8^{(1/2)}] = 0.40 \pm 0.10 \tag{3}$$

$$[g_{27}^{(1/2)}] = (3.2 \pm 0.8) \times 10^{-2} \tag{4}$$

$$[g_{27}^{(3/2)}] = 0.17 \tag{5}$$

It can be seen from equations (2) and (5) that the $|\Delta I| = 3/2$ amplitude is well understood. However, the $|\Delta I| = 1/2$ amplitude in equations (3) and (4) disagrees with the experimental value in equation (1) by more than an order of magnitude.

The QCD corrections have been calculated (Pich and de Rafael, 1991; Pich, 1988) to $O(\alpha_s^2)$

$$f(\alpha_s) = 1 + \frac{117501}{4840}\left(\frac{\alpha_s}{\pi}\right) + 470.72\left(\frac{\alpha_s}{\pi}\right)^2 \tag{6}$$

where $f(\alpha_s)$ represents the gluonic corrections to the two-point function

$$\Psi_{66}(q^2) = i\int d^4x\, e^{iqx}\langle 0 | T(Q_6(x)Q_6^+(0)) | 0\rangle \tag{7}$$

due to the so-called "Penguin diagrams." If we use the value of α_s at a few GeV

$$0.19 \le \alpha_s \le 0.31 \tag{8}$$

or

$$0.060 \le \alpha_s/\pi \le 0.10 \tag{9}$$

one sees that the series for $f(\alpha_s)$ is diverging at the lower limit of (9). The first reaction is to throw up one's hands and say that the series

$$f(\alpha_s) = 1 + 1.46 + 1.69 + \cdots \tag{10}$$

explodes and thus is meaningless. However, we will show that the series in equation (6) is, in fact, meaningful and can be summed by using Padé approximants (PA). We have used PA recently to estimate the next unknown term and the sum of the series (full Padé) in many examples in QED, QCD, atomic physics, and statistical physics (as well as applied mathematics) (Samuel *et al.*, 1993, 1994, 1995a,b; Samuel and Li, 1994a–c; Samuel and Druger, 1995; Samuel, 1995).

From equations (1), (3), and (4) we see that we need an enhancement of 12.8 ± 3.2. From equation (6) the series to be analyzed is given by

$$S = \sum S_n X^n \tag{11}$$

where $S_0 = 1$, $S_1 = 24.3$, $S_2 = 470.7$, and $X = \alpha_s/\pi$, where X is given in

Table I. Padé Estimates for the "Sum" of the Series in Equation (6)

$X = \alpha_s/\pi$	[1/2]	[2/1]
0.06	−12.9	−11.4
0.07	−5.17	−4.76
0.08	−3.40	−3.13
0.09	−2.63	−2.40
0.10	−2.21	−1.98

equation (9). We now estimate the next term in this series using our method of Padé approximants. The [*N*/*M*] PA is the ratio of two polynomials R_N and Q_M, where R_N is of degree *N* and Q_M is of degree *M*.

From the [1/1] we obtain

$$S_3 = 9118 \tag{12}$$

while from the [0/2] we get

$$S_3 = 8527 \tag{13}$$

We take the average of equations (12) and (13),

$$S_3 = 8800 \pm 600 \tag{14}$$

Now using S_0, S_1, S_2, and S_3 we construct the [1/2] PA and the [2/1] PA. The results are given in Table I. It can be seen that we can obtain large enhancement for reasonable values of α_s.

It may seem strange that the "sum" of a series whose terms are all positive could be negative. To understand this, consider the simple series

$$S = \frac{1}{1 - x} = 1 + x + x^2 + \cdots \tag{15}$$

If $x = 0.95$, $S = 20$, but one needs a large number of terms to obtain this result. However, if $x = 1.05$, all the terms are positive, but the "sum" $S = -20$. In both cases, however, from the first three terms, if one constructs the PA, one obtains $(1 - x)^{-1}$ and the exact result. This is what we believe is happening with Ψ_{66}.

Thus we see that our PA method allows us to sum a perturbation series which appears to be "blowing up" and enables us to understand the large enhancement of the $|\Delta I| = 1/2$ amplitude within the Standard Model.

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